Analytical Theory for Charged Particles with Saddle-Shaped Elements in Solution of a Symmetric Electrolyte

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Charged saddle-like shaped objects immersed in electrolyte solution are found in a variety of systems and include bent polyelectrolyte chains, deformed liquid crystalline lamellae, ring-like surfactant vesicles, curved wormlike micelles and their branching portions, biological membranes, protein molecules of complex shape, etc. The success of molecular thermodynamic models for such complex ionic systems depends crucially on our ability to describe the electrostatic free energy. Existing analytical models describe charged particles of lamellar, cylindrical and spherical shape. For saddle-like structures, to the best of our knowledge there has been no analytical formula for the electrostatic free energy. We propose an approximate analytical solution of the linearized Poisson-Boltzmann equation for charged torus's element immersed in symmetric electrolyte that is in excellent agreement with the results of a numerical solution. A simple correlation is found between the electrostatic potential of a saddle-like surface and its local curvature. We obtain simple analytical expression for the electrostatic free energy of a saddle-like structure. Our theory is tested by calculating the electrostatic contribution to the persistent length of a polyelectrolyte chain and by modeling the free energy for branching wormlike aggregate of ionic surfactant. We show that our theory adequately describes micellar branching and predicts correctly the shape of the experimental persistent length-salinity curve. We expect that apart from systems considered in this work, our simple electrostatics may be useful in many other engineering applications. For financial support the authors are grateful to RFBR, project #05-03-33267, and to Federal program "Leading scientific schools of Russia", project # 676.2003.3.